Probability Exercises

CHAPTER 1

Combinatorics

1.1. Problems

(1) Suppose a License plate must consist of 7 numbers of letter. How many license plates are there if
  (a) there can only be letters?
  (b) the first three places are numbers and the last four are letters?
  (c) the first three places are numbers and the last four are letters, but there can not be any repetitions in the same license plate?
(2) A school of 50 students has awards for the top math, english, history and science student in the school
  (a) How many ways can these awards be given if each student can only win one award?
  (b) How many ways can these awards be given if students can win multiple awards?
(3) An iPhone password can be made up of any 4 digit combination.
  (a) How many different passwords are possible?
  (b) How many are possible if all the digits are odd?
  (c) How many can be made in which all digits are different or all digits are the same?
(4) There is a class of 25 people made up of 11 guys and 14 girls.
  (a) How many ways are there to make a committee of 5 people?
  (b) How many ways are there to pick a committee of 5 of all girls?
  (c) How many ways are there to pick a committee of 3 girls and 2 guys?
(5) If a student council contains 10 people, how many ways are there to elect a president, a vice president, and a 3 person prom committee from the group of 10 students?
(6) Suppose you are organizing your textbooks on a book shelf. You have three chemistry books, 5 math books, 2 history books and 3 english books.
  (a) How many ways can you order the textbooks if you must have math books first, english books second, chemistry third, and history fourth?
  (b) How many ways can you order the books if each subject must be ordered together?
(7) You buy a Powerball lottery ticket. You choose 5 numbers between 1 and 59 (picked on white balls) and one number between 1 and 35 (picked on a red ball).How many ways can you
  (a) win the jackpot (guess all the numbers correctly)?
  (b) match all the white balls but not the red ball?
  (c) match 3 white balls and the red ball?
(8) A couple wants to invite their friends to be in their wedding party. The groom has 8 possible groomsmen and the bride has 11 possible bridesmaids. The wedding party will consist of 5 groomsman and 5 bridesmaids.
  (a) How many wedding party's are possible?
  (b) Suppose that two of the possible groomsmen are feuding and will only accept an invitation if the other one is not going. How many wedding party's are possible?

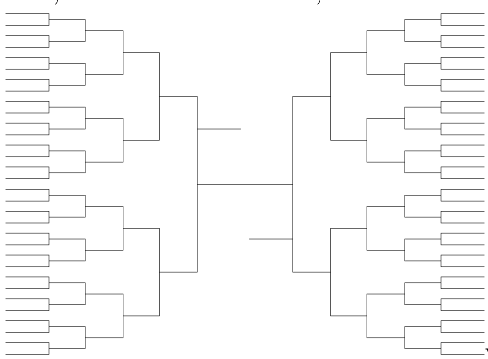
(c) Suppose that two of the possible bridesmaids are feuding and will only accept an invitation if the other one is not going. How many wedding party's are possible?

(d) Suppose that one possible groosman and one possible woman refuse to serve together. How many wedding party's are possible?

(9) There are 52 cards in a standard deck of playing cards. The poker hand is consists of five cards. How many poker hands are there?

(10) There are 30 people in a communications class. Each student must have a conversation with each student in the class for a project. How many total convesations will there be?

(11) Suppose a college basketball tournament consists of 64 teams playing head to head in a knock-out style tournament. There are 6 rounds, the round of 64, round of 32, round of 16, round of 8, the final four teams, and the finals. Suppose you are filling out a bracket such as this



which specifies which teams will win each game in each round. How many possible brackets can you make?

(12) You have eight distinct pieces of food. You want to choose three for breakfast, two for lunch, and three for dinner. How many ways to do that?

CHAPTER 2

The probability set up

(1) Suppose a box contains 3 balls : 1 red, 1 green, and 1 blue
  (a) Consider an experiment that consists of taking 1 ball from the box and then replacing it in the box and drawing a second ball from the box. List all possible outcomes.
  (b) Consider an experiment that consists of taking 1 ball from the box and then drawing a second ball from the box without replacing the first. List all possible outcomes.
(2) Suppose that A and B are mutually exclusive events for which $P(A) = .3$ and $P(B) = .5$.
  (a) What is the probability that A occurs but B does not?
  (b) What is the probability that neither A nor B occurs?
(3) Forty percent of college students from a certain college are members of neither an academic club nor a greek organization. Fifty percent are members of academic clubs while thirty percent are members of a greek organization. Suppose a student is chosen at random, what is the probability that this students is a member
  (a) of an academic club or a greek organization?
  (b) of an academic club and a greek organization?
(4) In City, 60% of the households subscribe to newspaper A, 50% to newspaper B, 40% to newspaper C, 30% to A and B, 20% to B and C, and 10% to A and C, but none subscribe to all three. (Hint: Draw a Venn diagram)
  (a) What percentage subscribe to exactly one newspaper?
  (b) What percentage subscribe to at most one newspaper?
(5) There are 52 cards in a standard deck of playing cards. The *poker hand* is consists of five cards. There are 4 *suits*: heats, spades, diamonds, and clubs ($\heartsuit\spadesuit\diamondsuit\clubsuit$). The suit's diamonds and clubs are red while clubs and spades are black. In each suit there are 13 *ranks*: The Ace card, the numbers $2, 3 \ldots, 10$, the face cards, Jack, Queen and King. Find the probability of randomly drawing the following poker hands.
  (a) All red cards?
  (b) Exactly two 10's and exactly three aces?
  (c) all face cards or no face cards?
(6) Find the probability of randomly drawing the following poker hands.
  (a) A *one pair,* which consists of two cards of the same rank and three other distinct ranks. (e.g. 22Q59)
  (b) A *two pair*, which constists of two cards of the same rank, two cards of another rank, and another card of yet another rank. (e.g. JJ779)
  (c) A *three of a kind*, which consists of a three cards of the same rank, and two others of distinct rank. (e.g. 4449K)
  (d) A *flush*, which consists of all five cards of the same suit. (e.g. HHHH, SSSS, DDDD, or CCCC)
  (e) A *full house*, which consists of a two pair and a three of a kind. (e.g. 88844) (Hint: Note that 88844 is a different hand than a 44488)

(7) Suppose a standard deck of cards is modifed with the additional rank of *Super King* and the additional suit of *Swords* so now each card has one of 14 ranks and one of 5 suits.
   (a) If a card is selected at random, what is the probability that it's the *Super King of Swords*.
   (b) What's the probability of getting a six card hand with exactly three pairs (two cards of one rank and two cards of another rank and two cards of yet another rank, e.g. 7,7,2,2,J,J) ?
   (c) What's the probability of getting a six card hand which constists of three cards of the same rank, two cards of another rank, and another card of yet another rank. (e.g. 3,3,3,A,A,7)?
   (d) What's the probability of getting a six card hand with exactly 2 pairs, and 2 singles (two cards of one rank and 1 card of another rank and 1 card of yet another rank, e.g. 7,7,2,2,J,K) ?

(8) A pair of fair dice is rolled. What is the probability that the first die lands on a strictly higher value than the second die.

(9) There are 8 students in a class. What is the probability that at least two students share a common birthday month?

(10) Nine balls are randomly withdrawn from an urn that contains 10 blue, 12 red, and 15 green balls. What is the probability that
   (a) 2 blue, 5 red, and 2 green balls are withdrawn
   (b) at least 2 blue balls are withdrawn.

(11) Suppose 4 valedictorians (from different high schools) were all accepted to the 8 Ivy League universities. What is the probability that they each choose to go to a different Ivy League university?

(12) Let S be a sample space with probability $\mathbb{P}$. Let E, F be any events in S. Using the Axioms of Probability or Proposition 1 from the Lecture notes (Section 2.2),
   (a) Show that $\mathbb{P}(E \cap F^c) = \mathbb{P}(E) - \mathbb{P}(E \cap F)$:
   (b) Show *Bonferroni's inequality*: $\mathbb{P}(E \cap F) \geq \mathbb{P}(E) + \mathbb{P}(F) - 1$:

CHAPTER 3

Independence

(1) Let A and B be two independent events with $P(A) = .4$ and $P(A \cup B) = .64$. What is $P(B)$?

(2) In a class, there are 4 male math majors, 6 female math majors, and 6 male actuarial science majors. How many actuarial science girls must be present in the class if sex and major are independent when choosing a student selected at random?

(3) An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44. Calculate the number of blue balls in the second urn.

(4) Using only the definition of independence and any properties you already know about events/sets and probability, prove that if E and F are independent then E^c and F^c must also be independent.

CHAPTER 4

Conditional Probability

(1) Two dice are rolled. Let $A = \{\text{sum of two dice equals 3}\}$, $B = \{\text{sum of two dice equals 7 }\}$, and $C = \{\text{at least one of the dice shows a 1}\}$.
  (a) What is $\mathbb{P}(A \mid C)$?
  (b) What is $\mathbb{P}(B \mid C)$?
  (c) Are A and C indepedent? What about B and C?

(2) Suppose you roll a two standard, fair, 6-sided dice. What is the probability that the sum is at least 9 given that you rolled at least one 6?

(3) Suppose that Annabelle and Bobby each draw 13 cards from a standard deck of 52. Given that Sarah has exactly two aces, what is the probability that Bobby has exactly one ace?

(4) Color blindness is a sex-linked condition, and 5% of men and 0.25% of women are color blind. The population of the United States is 51% female. What is the probability that a color-blind American is a man?

(5) Suppose that two factories supply light bulbs to the market. Factory X's bulbs work for over 5000 hours in 99% of cases, whereas factory Y's bulbs work for over 5000 hours in 95% of cases. It is known that factory X supplies 60% of the total bulbs available.
  (a) What is the chance that a purchased bulb will work for longer than 5000 hours?
  (b) Given that a lightbulb works for more than 5000 hours, what is the probability that it came from factory Y?
  (c) Given that a lightbulb work does not work for more than 5000 hours, what is the probability that it came from factory X?

(6) A factory production line is manufacturing bolts using three machines, A, B and C. Of the total output, machine A is responsible for 25%, machine B for 35% and machine C for the rest. It is known from previous experience with the machines that 5% of the output from machine A is defective, 4% from machine B and 2% from machine C. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from Machine A?

(7) A multiple choice exam has 4 choices for each question. A student has studied enough so that the probability they will know the answer to a question is 0.5, the probability that they will be able to eliminate one choice is 0.25, otherwise all 4 choices seem equally plausible. If they know the answer they will get the question right. If not they have to guess from the 3 or 4 choices. As the teacher you want the test to measure what the student knows. If the student answers a question correctly what's the probability they knew the answer?

(8) A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not actually present. One percent of the population actually has the disease. Calculate the probability that a person actually has the disease given that the test indicates the presence of the disease.

CHAPTER 5

Random Variables

(1) Two balls are chosen randomly from an urn containing 8 white balls, 4 black, and 2 orange balls. Supose that we win \$2 for each black ball selected and we lose \$1 for each white ball selected. Let X denote our winnings.
  (a) What are the possible values of X?
  (b) What are the probabilities associated to each value?

(2) A card is drawn at random from a standard deck of playing cards. If it is a heart, you win \$1. If it is a diamond, you have to pay \$2. If it is any other card, you win \$3. What is the expected value of your winnings?

(3) The game of roulette consists of a small ball and a wheel with 38 numbered pockets around the edge that includes the numbers $1-36, 0$ and 00. As the wheel is spun, the ball bounces around randomly until it settles down in one of the pockets.
  (a) Suppose you bet \$1 on a single number and random variable X represents the (monetary) outcome (the money you win or lose). If the bet wins, the payoff is \$35 and you get your money back. If you lose the bet then you lose your \$1. What is the expected profit on a 1 dollar bet ?
  (b) Suppose you bet \$1 on the numbers $1-18$ and random variable X represents the (monetary) outcome (the money you win or lose). If the bet wins, the payoff is \$1 and you get your money back. If you lose the bet then you lose your \$1. What is the expected profit on a 1 dollar bet ?

(4) An insurance company finds that Mark has a 8% chance of getting into a car accident in the next year. If Mark has any kind of accident then the company guarantees to pay him \$10,000. The company has decided to charge Mark a \$200 premium for this one year insurance policy.
  (a) Let X be the amount profit or loss from this insurance policy in the next year for the insurance company. Find $\mathbb{E}X$, the expected return for the Insurance company? Should the insurance company charge more or less on it's premium?
  (b) What amount should the insurance company charge Mark in order to guarantee an expected return of \$100?

(5) A random variable X has the following probability mass function: $p(0) = \frac{1}{3}$, $p(1) = \frac{1}{6}$, $p(2) = \frac{1}{4}$, $p(3) = \frac{1}{4}$. Find its expected value, variance, and standard deviation.

(6) Suppose X is a random variable such that $\mathbb{E}[X] = 50$ and $\mathrm{Var}(X) = 12$. Calculate the following quantities.
  (a) $\mathbb{E}\left[X^2\right]$
  (b) $\mathbb{E}[3X+2]$
  (c) $\mathbb{E}\left[(X+2)^2\right]$
  (d) $\mathrm{Var}[-X]$
  (e) $SD(2X)$.

(7) Does there exists a random variable X such that $\mathbb{E}[X] = 4$ and $\mathbb{E}\left[X^2\right] = 10$? Why or why not ? (Hint: Look at its variance)

(8) Let X be the total number of text messages a random college student at Big State University receives in a year. Suppose you are given that the pmf of X is of the form

$$p_X(n) = \frac{c}{2^n}, \text{for } n = 0, 1, 2, 3, \ldots.$$

What must the value of c have to be if p_X is indeed a pmf of X? (Hint: Recall from Calculus 2 that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ whenever $0 < x < 1$)

(9) Let X be a discrete random variable with range given by $X \in \{1, 2, 3, \ldots\}$. Suppose the pmf of X is given by

$$p_X(n) = \frac{1}{2^n} \text{ for } n = 1, 2, 3, \ldots.$$

(a) Give a sketch of the graph of the CDF of X.
(b) What is the value of $F_X(4)$.
(c) Use the CDF to find $\mathbb{P}(X > 4)$.

(10) Suppose the CDF of X is given by

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{3} & -1 \leq x < 1, \\ \frac{x}{3} & 1 \leq x < 2 \\ 1 & x \geq 2. \end{cases}$$

(a) Plot this CDF.
(b) Find $\mathbb{P}(X > 1)$.
(c) Find $\mathbb{P}(X = 1)$.
(d) Find $\mathbb{P}(X = 2)$.

CHAPTER 6

Some Discrete Distributions

(1) A UConn student claims that she can tell Dairy Bar ice cream from Friendly's ice cream. As a test, she is given ten samples of ice cream (each sample is either from the Dairy Bar or friendly's) and asked to identify each one. She is right eight times. What is the probability that she would be right exactly eight times if she guessed randomly for each sample?

(2) A Pharmaceutical company conducted a study on a new drug that is supposed to treat patients suffering from a certain disease. The study concluded that the drug did not help 25% of those who participated in the study. What is the probability that of 6 randomly selected patients, 4 will recover?

(3) 20% of all students are left-handed. A class of size 20 meets in a room with 18 right-handed desks and 5 left-handed desks. What is the probability that every student will have a suitable desk?

(4) A ball is drawn from an urn containing 4 blue and 5 red balls. After the ball is drawn, it is replaced and another ball is drawn. Suppose this process is done 7 times.
   (a) What is the probability that exactly 2 red balls were drawn in the 7 draws?
   (b) What is the probability that at least 3 blue balls were drawn in the 7 draws?

(5) The expected number of typos on a page of the new Harry Potter book is .2. What is the probability that the next page you read contains
   (a) 0 typos ?
   (b) 2 or more typos?
   (c) Explain what your assumptions you used.

(6) The monthly average number of car crashes in Storrs, CT is 3.5. What is the probability that there will be
   (a) at least 2 accidents in the next month?
   (b) at most 1 accident in the next month?
   (c) Explain what your assumptions you used.

(7) Suppose that the average number of burglaries in New York City in a week is 2.2. Approximate the probability that there will be
   (a) no burglaries in the next week;
   (b) at least 2 burglaries in the next week.

(8) The number of accidents per working week in a particular shipyard is Poisson distributed with mean 0.5. Find the probability that:
   (a) In a particular week there will be at least 2 accidents.
   (b) In a particular two week period there will be exactly 5 accidents.
   (c) In a particular month (i.e. 4 week period) there will be exactly 2 accidents.

(9) Jennifer is baking cookies. She mixes 400 raisins and 600 chocolate chips into her cookie dough and ends up with 500 cookies.
   (a) Find the probability that a randomly picked cookie will have three raisins in it.
   (b) Find the probability that a randomly picked cookie will have at least one chocolate chip in it.

(c) Find the probability that a randomly picked cookie will have no more than two bits in it (a bit is either a raisin or a chocolate chip).

(10) Chevy has three factories that produces their car called *Camaro*. The average number of factory defects per Camaro is2.2 when built by the first factory, 4 when built by the second factory and 1.5 when built by the third factory. Suppose you buy a brand new Camaro from your local Chevy dealer. If your Camaro is equally likely to be typed by either factories, approximate the probability that it will have no defects. Assume factory defects per Camaro is Poisson distributed. (Hint: Law of total probability)

(11) A roulette wheel has 38 numbers on it: the numbers 0 through 36 and a 00. Suppose that Lauren always bets that the outcome will be a number between 1 and 18 (including 1 and 18).

(a) What is the probability that Lauren will lose her first 6 bets.

(b) What is the probability that Lauren will first win on her sixth bet?

(c) What is the expected number of bets until her first win?

CHAPTER 7

Continuous distributions

(1) Let X be a random variable with probability density function

$$f(x)=\begin{cases} cx\,(5-x) & 0\le x\le 5\\ 0 & \text{otherwise}\end{cases}$$

(a) What is the value of c?
(b) What is the cumulative distribution function of X? That is, find $F_X(x)=\mathbb{P}(X\le x)$.
(c) Use your answer in part (b) to find $\mathbb{P}(2\le X\le 3)$.
(d) What is $\mathbb{E}[X]$?
(e) What is $\mathrm{Var}(X)$?

(2) UNH students have designed the new u-phone. They have determined that the lifetime of a U-Phone is given by the random variable X (measured in hours), with probability density function

$$f(x)=\begin{cases} \frac{10}{x^2} & x\ge 10\\ 0 & x\le 10\end{cases}.$$

(a) Find the probability that the U-phone will last more than 20 hours?
(b) What is the cumulative distribution function of X? That is, find $F_X(x)=\mathbb{P}(X\le x)$.
(c) Use part (b) to help you find $\mathbb{P}(X\ge 35)$?

(3) Suppose the random variable X has a density function

$$f(x)=\begin{cases} \frac{2}{x^2} & x>2,\\ 0 & x\le 2.\end{cases}$$

Compute $\mathbb{E}[X]$.

(4) An insurance company insures a large number of homes. The insured value, X, of a randomly selected home is assumed to follow a distribution with density function

$$f(x)=\begin{cases} \frac{3}{x^4} & x>1,\\ 0 & \text{, otherwise.}\end{cases}$$

Given that a randomly selected home is insured for at least 1.5, calculate the probability that it is insured for less than 2.

(5) The density function of X is given by

$$f(x)=\begin{cases} a+bx^2 & 0\le x\le 1\\ 0 & \text{otherwise.}\end{cases}$$

If $\mathbb{E}[X]=\frac{7}{10}$, find a and b.

(6) Let X be a random variable with density function

$$f(x) = \begin{cases} \frac{1}{a-1} & 1 < x < a \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that $\mathbb{E}[X] = 6\mathrm{Var}(X)$. Find the value of a.

(7) Suppose you order a pizza from your favorite pizzaria at 7:00 pm, knowing that the time it takes for your pizza to be ready is uniformly distributed between 7:00 pm and 7:30 pm.

(a) What is the probability that you will have to wait longer than 10 minutes for your pizza?

(b) If at 7:15pm, the pizza has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

(8) Suppose an insurance policy offers Toy Insurance. The rules of the **benefit paid** under this policy are the following: it reimburses a loss up to a **benefit** limit of only 10 dollars, otherwise it pays full amount of the loss. The insurance company calculates that the policyholder's toy **loss**, Y, follows a distribution with density function given by:

$$f_Y(y) = \begin{cases} \frac{2}{y^2} & \text{for } y > 1 \\ 0 & \text{otherwise.} \end{cases}$$

Let X be the **benefit paid** under this insurance policy. What is the expected value of X? (The question is **NOT** asking for the expected value of the loss Y)

CHAPTER 8

Normal Distributions

(1) Suppose X is a normally distributed random variable with $\mu = 10$ and $\sigma^2 = 36$. Find
   (a) $\mathbb{P}(X > 5)$,
   (b) $\mathbb{P}(4 < X < 16)$,
   (c) $\mathbb{P}(X < 8)$.
(2) The height of maple trees at age 10 are estimated to be normally distributed with mean 200 cm and variance 64 cm. What is the probability a maple tree at age 10 grows more than 210cm?
(3) The peak temperature T, in degrees Fahrenheit, on a July day in Antarctica is a Normal random variable with a variance of 225. With probability .5, the temperature T exceeds 10 degrees.
   (a) What is $\mathbb{P}(T > 32)$, the probability the temperature is above freezing?
   (b) What is $\mathbb{P}(T < 0)$?
(4) The salaries of UConn professors is approximately normally distributed. Suppose you know that 33 percent of professors earn less than \$80,000. Also 33 percent earn more than \$120,000.
   (a) What is the probability that a UConn professor makes more than \$100,000?
   (b) What is the probability that a UConn professor makes between \$70,000 and \$80,000?
(5) Suppose X is a normal random variable with mean 5. If $\mathbb{P}(X > 0) = .8888$, approximately what is $Var(X)$?
(6) The shoe size of a UConn basketball player is normally distributed with mean 12 inches and variance 4 inches. Ten percent of all UConn basketball players have a shoe size greater than c inches. Find the value of c.

CHAPTER 9

Normal Approximation to the binomial

(1) Suppose that we roll 2 dice 180 times. Let E be the event that we roll two fives no more than once.
   (a) Find the exact probability of E.
   (b) Approximate $\mathbb{P}(E)$ using the normal distribution.
   (c) Approximate $\mathbb{P}(E)$ using the Poisson distribution.

CHAPTER 10

Some continuous distributions

(1) Suppose that the time required to replace a car's windshield can be represented by an exponentially distributed random variable with parameter $\lambda = \frac{1}{2}$.
  (a) What is the probability that it will take at least 3 hours to replace a windshield?
  (b) What is the probability that it will take at least 5 hours to replace a windshield given that it hasn't been finished after 2 hours?

(2) The number of years a u-phone functions is exponentially distributed with parameter $\lambda = \frac{1}{8}$. If Pat buys a used u-phone, what is the probability that it will be working after an additional 8 years?

(3) Suppose that the time (in minutes) required to check out a book at the library can be represented by an exponentially distributed random variable with parameter $\lambda = \frac{2}{11}$.
  (a) What is the probability that it will take at least 5 minutes to check out a book?
  (b) What is the probability that it will take at least 11 minutes to check out a book given that you've already waited for 6 minutes?

(4) Let X be an exponential random variable with mean $\mathbb{E}[X] = 1$. Define a new random variable $Y = e^X$. Find the p.d.f. of Y, $f_Y(y)$, and then use the p.d.f of Y to compute the mean of Y.

(5) Suppose that X has an exponential distribution with parameter $\lambda = 1$. Let $c > 0$. Show that $Y = \frac{X}{c}$ is exponential with parameter $\lambda = c$.

(6) Let X be a uniform random variable over $(0, 1)$. Define a new random variable $Y = e^X$. Find the probability density function of Y, $f_Y(y)$.

(7) Suppose an amazon box always has a square base with height twice as much as the length of its base. Suppose it is known that the side length of the square base is given by a random variable X in inches with PDF given by

$$f_X(x) = \begin{cases} \frac{1}{9}x^2 & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}.$$

Find the PDF of Y, the volume of the box and use the PDF to calculate the mean volume of an Amazon box.

CHAPTER 11

Multivariate distributions

(1) Suppose that 2 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let X equal 1 if the first ball selected is white and zero otherwise. Let Y equal 1 if the second ball selected is white and zero otherwise. Find the probability mass function of X, Y.

(2) Suppose you roll two fair dice. Find the probability mass function of X and Y, where X is the largest value obtained on any die, and Y is the sum of the values.

(3) Suppose the joint density function of X and Y is $f(x,y) = \frac{1}{4}$ for $0 < x < 2$ and $0 < y < 2$.
  (a) Calculate $\mathbb{P}\left(\frac{1}{2} < X < 1, \frac{2}{3} < Y < \frac{4}{3}\right)$.
  (b) Calculate $\mathbb{P}(XY < 2)$.
  (c) Calculate the marginal distributions $f_X(x)$ adn $f_Y(y)$.

(4) The joint probability density function of X and Y is given by

$$f(x,y) = e^{-(x+y)}, \quad 0 \leq x < \infty, 0 \leq y < \infty.$$

Find $\mathbb{P}(X < Y)$.

(5) Suppose X and Y are independent random variables and that X is exponential with $\lambda = \frac{1}{4}$ and Y is uniform on $(2,5)$. Calculate the probability that $2X + Y < 8$.

(6) Consider X and Y given by the joint density

$$f(x,y) = \begin{cases} 10x^2 y & 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

  (a) Find the marginal pdfs, $f_X(x)$ and $f_Y(x)$
  (b) Are X and Y independent random variables?
  (c) Find $\mathbb{P}\left(Y \leq \frac{X}{2}\right)$
  (d) Find $\mathbb{P}\left(Y \leq \frac{X}{4} \mid Y \leq \frac{X}{2}\right)$
  (e) Find $\mathbb{E}[X]$.

(7) Consider X and Y given by the joint density

$$f(x,y) = \begin{cases} 4xy & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

  (a) Find the joint pdf's, f_X and f_Y.
  (b) Are X and Y independent?
  (c) Find $\mathbb{E}Y$.

(8) Consider X, Y given by the joint pdf

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Are X and Y independent random variables?

(9) Suppose that that gross weekly ticket sales for UConn basketball games are normally distributed with mean $\$2,200,000$ and standard deviation $\$230,000$. What is the probability that the total gross ticket sales over the next two weeks exceeds $\$4,600,000$? What assumption did you use?

(10) Suppose the joint density function of the random variable X_1 and X_2 are

$$f(x_1,x_2)=\begin{cases}4x_1x_2 & 0<x_1<1, 0<x_2<1\\ 0 & \text{otherwise.}\end{cases}$$

Let $Y_1=2X_1+X_2$ and $Y_2=X_1-3X_2$. What is the joint density function of Y_1 and Y_2?

(11) Suppose the joint density function of the random variable X_1 and X_2 are

$$f(x_1,x_2)=\begin{cases}\frac{3}{2}\left(x_1^2+x_2^2\right) & 0<x_1<1, 0<x_2<1\\ 0 & \text{otherwise.}\end{cases}$$

Let $Y_1=X_1-2x_2$ and $Y_2=2X_1+3X_2$. What is the joint density function of Y_1 and Y_2?

CHAPTER 12

Expectations

(1) Suppose the joint distribution for X and Y is given by the joint probability mass function shown below:

$Y \backslash X$	0	1
0	0	.3
1	.5	.2

. Calculate $\mathbb{E}[XY], \mathbb{E}[X], \mathbb{E}[Y]$.

(2) Let X and Y be random variables whose joint probability density function is given by

$$f(x,y) = \begin{cases} x+y & 0<x<1, 0<y<1 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate $\mathbb{E}[XY], \mathbb{E}X$, and $\mathbb{E}Y$.

(3) Let X be normally distributed with mean 1 and variance 9. Let Y be exponentially distributed with $\lambda = 2$. Suppose X and Y are independent. Find $\mathbb{E}\left[(X-1)^2 Y\right]$. (Hint: Use a properties about expectations)

(4) Suppose $X \sim \text{Bern}\left(\frac{1}{2}\right)$ and $Y \sim \text{Exp}(2)$ are independent then calculate $\mathbb{E}\left[e^{XY}\right]$.

(5) Suppose the joint distribution for X and Y is given by the joint probability mass function shown below:

$Y \backslash X$	0	1
0	0	.3
1	.5	.2

(a) Calculate the covariance of X and Y.
(b) Calculate $\text{Var}(X)$ and $\text{Var}(Y)$.
(c) Calculate $\rho(X,Y)$.

(6) Let X and Y be random variable whose joint probability density function is given by

$$f(x,y) = \begin{cases} x+y & 0<x<1, 0<y<1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Calculate the covariance of X and Y.
(b) Calculate $\text{Var}(X)$ and $\text{Var}(Y)$.
(c) Calculate $\rho(X,Y)$.

CHAPTER 13

Moment generating functions

(1) Suppose that you have a fair 4-sided die, and let X be the random variable representing the value of the number rolled.
 (a) Write down the moment generating function for X.
 (b) Use this moment generating function to compute the first and second moments of X.

(2) Let X be a random variable whose probability density function is given by

$$f_X(x) = \begin{cases} e^{-2x} + \frac{1}{2}e^{-x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

 (a) Write down the moment generating function for X.
 (b) Use this moment generating function to compute the first and second moments of X.

(3) Suppose X and Y are Poisson independent random variables with parameters λ_x, λ_y, respectively. Find the distribution of $X+Y$.

(4) True or False? Suppose $X \sim \operatorname{Exp}(\lambda_x)$ and $Y \sim \operatorname{Exp}(\lambda_y)$ with parameters $\lambda_x, \lambda_y > 0$ and suppose X, Y are independent. Is $X + Y \sim \operatorname{Exp}(\lambda_x + \lambda_y)$?

(5) Suppose that a mathematician determines that the revenue the UConn Dairy Bar makes in a week is a random variable, X, with moment generating function

$$M_X(t) = \frac{1}{(1-2500t)^4}.$$

Calculate the standard deviation of the revenue the UConn Dairy bar makes in a week.

(6) Let X and Y be two independent random variables with respective moment generating functions

$$m_X(t) = \frac{1}{1-5t}, \quad \text{if } t < \frac{1}{5}, \quad m_Y(t) = \frac{1}{(1-5t)^2}, \quad \text{if } t < \frac{1}{5}.$$

Find $\mathbb{E}(X+Y)^2$.

(7) Suppose $X \sim \operatorname{Exp}(2), Y \sim \operatorname{Bern}\left(\frac{1}{2}\right), Z \sim \operatorname{Exp}(1)$. Suppose X, Y, Z are independent and define the random variable

$$W = X + YZ.$$

Compute the moment generating function of W and find the distribution of W exactly. (Hint: Simplify as much as possible and your answer will be one of the known distributions in the distribution table.)

CHAPTER 14

Limit Laws

(1) In a 162-game season, find the approximate probability that a team with a 0.5 chance of winning will win at least 87 games.
(2) An individual students MATH 3160 Final exam score at UConn is a random variable with mean 75 and variance 25, How many students would have to take the examination to ensure with probability at least .9 that the class average would be within 5 of 75?
(3) Let $X_1, X_2, \ldots, X_{100}$ be independent exponential random variables with parameter $\lambda = 1$. Use the central limit theorem to approximate
$$\mathbb{P}\left(\sum_{i=1}^{100} X_i > 90\right).$$
(4) Suppose an insurance company has 10,000 automobile policy holders. The expected yearly claim per policy holder is \$240, with a standard deviation of \$800. Approximate the probability that the total yearly claim is greater than \$2,500,000.
(5) Suppose you are the only clerk at the UConn dairy bar. Suppose that the checkout time at the dairy bar has a mean of 5 minutes and a standard deviation of 2 minutes. Estimate the probability that a clerk will serve at least 36 customers during her 3-hour and a half shift.
(6) Shabazz Napier is a basketball player in the NBA. His expected number of points per game is 15 with a standard deviation of 5 points per game. The NBA season is 82 games long. Shabazz is guaranteed a ten million dollar raise next year if he can score a total of 1300 points this season. Approximate the probability that Shabazz will get a raise next season.